Testing the Scintillation Line Shape

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In current approach we adopted the usage of Generalized gamma function (GGF) as a model for the scintillation line shape. GGF was introduced and tested for the CTF data, i.e. for charge variable. In contrast to the charge variable, there is no theory behind the usage of GGF for Npmts variable. It worked for Npmts normalized, but the limits of applicability have never been tested. There were some tests with Borexino MC data with statistics of the level of 10^4 and because it performed much better than Gaussian, the GGF was a response function of choice for Be-7 analysis.

The question is whether GGF is a good "base distribution" for the pp fit. By “base distribution” I mean statistical part of the response shape. Some deviations from this shape are possible due to the varying PMTs number (this effect can be accounted for precisely if necessary) and due to the volume (geometric) effects. The last effect could be simulated with a good precision with MC. In principle, the “base distribution” is binomial. When dealing with real response one should adjust the “base distribution” width to take into account at least the additional smearing of the signal due to the above mentioned factors. There are also other factors, such as intrinsic line width and probably some others. The problem with binomial “base function” (or with its Poissonian approximation) is that its width is defined by the mean value. In case of Poisson the variance of the signal coincide with mean μ. Looking into the literature I found that scaled Poisson distribution is usually used in these cases. Its advantage is the possibility to adjust not only the mean value but also the variance of the signal. It has a very simple form:

$$f(x) = \frac{\mu^{xs}}{(xs)!} e^{-\mu},$$

and features two parameters, that could be evaluated using expected mean and variance.

I performed a set of tests with our standard GGF function and scaled Poisson (SP) function in order to see if the results are compatible. First, I've put monoenergetic source at the detector's center (apparently the binomual distribution was simulated). The results are presented in Table 1

Events	10^7
Gauss	31777.6/71
GGF	272.8/61
SP	39.4/61

Table 1: Results of the response function fit with 3 base functions. <Npm>=49.9.

Then I've used a toy MC that reproduces Borexino response function for monoenergetic particle. The results of fit of the MC response with different models are presented in Tables 3 (tests with different statistics for <Npm>=49.6) and 4 (test with statistics 10^7 in IV (factor 0.27 less in the FV) for different energies, close to what we have for ^{14}C). As tests show, the quality of approximation strongly depends on the statistics to be fit. GGF is still OK with $10^4 - 10^5$ events from the Po peak and SP provides no visible advantages. But there are deviations in shape if going to lower Npm and higher statistics. So, for C14 simulation with statistics approaching 10^7 in every bin, the use of proper response function is critical.

In Table I've calculated the fraction of events in tails of $N > \overline{N} + 1, 2, 3, 4\sigma$ (there is a small bias for gaussian because of the integer nature of variable, I've rounded the limits).

	1σ	2σ	3σ	4σ
Gauss	$1.5 \cdot 10^{-1}$	$2.10 \cdot 10^{-2}$	$1.2 \cdot 10^{-3}$	$2.5 \cdot 10^{-5}$
GGF	$1.5 \cdot 10^{-1}$	$2.45 \cdot 10^{-2}$	$1.9 \cdot 10^{-3}$	$7.8 \cdot 10^{-5}$
SP	$1.5 \cdot 10^{-1}$	$2.45 \cdot 10^{-2}$	$2.0 \cdot 10^{-3}$	$8.4 \cdot 10^{-5}$

Table 2: Fraction of events in tail for 2 base functions (<N>=49.9, source at center)

The results of fit with GGF and SP response function of our data are presented in

Table 5. The results of second cluster fit for ^{14}C and L.Y. are shown for comparison in Table 6 (this fit is apparently not sensitive to the response function, even Gaussian could be used due to the relatively low statistics in every bin).

Fit with GGF function produces systematically lower L.Y. and higher C14 content compared to fit with SP function. The values for C14 content obtained with second cluster data are closer to those of fit with SP function.

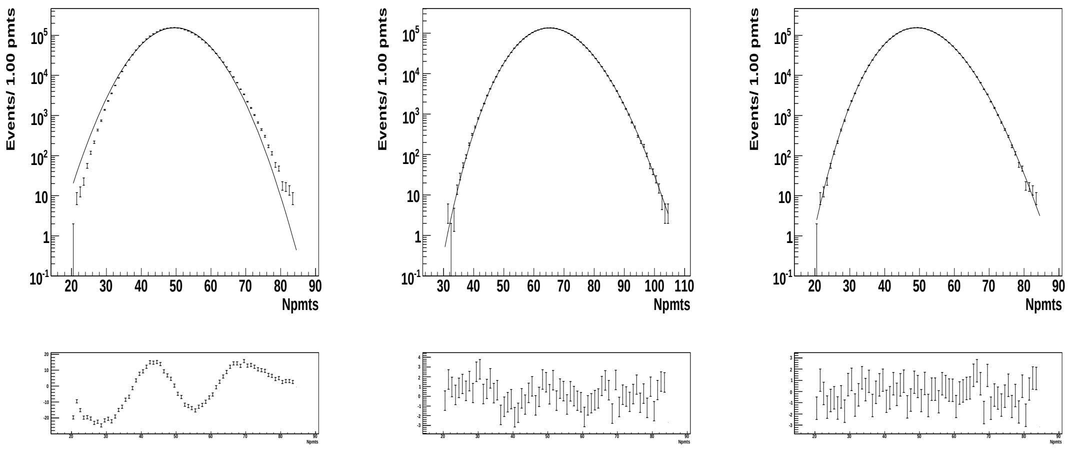

Figure 1: Fits of MC response function (10^7 event in IV, <Npm>=49.6) with Gaussian, GGF and SP models correspondingly.

Events	$10^4(\times0.27)$	$10^5(\times0.27)$	$10^6(\times0.27)$	$10^7(\times0.27)$
Gauss	65.6/61	141.5/61	852.8/61	8674.8/61
GGF	51.3/61	35.3/61	72.1/61	88.0/61
SP	51.2/61	34.9/61	59.2/61	59.3/61

Table 3: Results of the response function fit for different statistics with 3 base functions. <Npm>=49.6.

<Npmts>	33.4	49.6	65.7	143.4	143.4
Gauss	12715.3/56	8674.8/61	6059.5/76	2480.9/106	366.1/106
GGF	201.8/56	88.0/61	111.9/76	173.0/106	116.7/106
SP	62.2/56	59.3/61	80.9/76	155.0/106	111.8/106

Table 4: $10^7(\cdot0.27)$ statistics, last column is for $10^6(\cdot0.27)$ statistics

Period	response	L.Y.	^{14}C	χ^2/n.d.f.
9	GGF	491.0	39.1±1.2	157.9/162
	SP	493.4	38.2±1.1	156.3/162
10	GGF	482.9	42.4±1.5	176.0/157
	SP	486	41.1±1.4	175.6/157
11	GGF	481.5	43.4±1.6	155.8/152
	SP	484.9	42.0±1.4	156.6/152
12	GGF	483.4	41.9±1.2	202.6/150
	SP	486.6	40.7±1.1	205.6/150
9+10+11	GGF	486.5	41.2±0.8	179.1/158
	SP	489.6	40.0±0.8	178.6/158
9+10+11+12	GGF	481.5	41.3±0.5	201.6/157
	SP	484.2	40.3±0.6	204.1/157

Table 5: Results of fit with two response functions: "GGF" stays for standard method (generalized gamma), "SP" - scaled Poisson. Fit ranges correspond are adjusted in accordance with Livia's table.

response	L.Y.	^{14}C	χ^2/n.d.f.
GGF	496.2±4.6	39.8±0.9	38.6/37
SP	496.4±4.6	39.9±0.9	38.6/37

Table 6: Results of fit of the second cluster with two response functions, fit range is 40-80.